

Black Hole Macro-Quantumness

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Abstract

It is a common wisdom that properties of macroscopic bodies are well described by (semi)classical physics. As we have suggested [1–3] this wisdom is not applicable to black holes. Despite being macroscopic, black holes are quantum objects. They represent Bose-Einstein condensates of N -soft gravitons at the quantum critical point, where N Bogoliubov modes become gapless. As a result, physics governing arbitrarily-large black holes (e.g., of galactic size) is a quantum physics of the collective Bogoliubov modes. This fact introduces a new intrinsically-quantum corrections in form of $1/N$, as opposed to e^{-N} . These corrections are unaccounted by the usual semiclassical expansion in \hbar and cannot be recast in form of a quantum back-reaction to classical metric. Instead the metric itself becomes an approximate entity. These $1/N$ corrections abolish the presumed properties of black holes, such as non existence of hair, and are the key to nullifying the so-called information paradox.

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1 Essence of Macro-Quantumness

It is a common wisdom that the properties derived in idealized semi-classical treatment, such as, e.g., Hawking's exact thermality [5] [6] and absence of hair [7], must be well-applicable to the real macroscopic black holes. From the first glance, this sounds reasonable. After all, the common effective-field-theoretic sense tells us that for large objects all the microscopic quantum physics averages out in effective macroscopic characteristics, which are classical. When applying this reasoning to ordinary macroscopic objects such as planets, stars or galaxies, no apparent paradoxes or inconsistencies appear. For example, treating the earth as a semi-classical gravitating source gives a consistent picture.

In contrast, when applying the same common sense to realistic macroscopic black holes of finite mass, one ends up with puzzles and paradoxes, perhaps the most prominent being Hawking's information paradox [8]. The purpose of this short note is not to discuss the existing puzzles one by one, but instead to point out the misconception that underlies all of them. Namely, the quantum effects for the macroscopic black holes are much more important than what is suggested by straightforward application of semi-classical reasoning. This is the lesson from the recently-developed black hole quantum portrait [1–4]. In this respect there is nothing new in the present note, but we shall provide a sharper focus and specifically reiterate the key point that we believe sources the black hole mysteries. We would like to explain why it happens that some macroscopic bodies are more quantum than others.

The short answer is that despite being macroscopic, black holes are systems at the critical point of a *quantum* phase transition [3]. As a result, no matter how large and heavy, they can never be treated fully classically. Indeed the very nature of the phase transition as quantum requires a great amount of quantumness in the form of entanglement. Of course, for some aspects (e.g., large distance gravitational effects on probe bodies) the semi-classical treatment is fine, but is non-applicable for other aspects, such as information storage and processing.

In order to explain this profound difference, let us compare a black hole to an ordinary macroscopic object, e.g., a planet or a bucket of water. Of course the common property of all the macroscopic objects, that allows to treat them in long-distance regimes classically, is the large number of quantum constituents, N . Property, $N \gg 1$ is universally shared by all the macroscopic bodies of our interest.

For such objects we can define some quantum characteristics, such as, N (e.g., number of atoms in the bucket of water) and their *quantum* coupling strengths. However, in ordinary objects with size much bigger than the de-Broglie wave-lengths of the constituents, the coupling α_{ij} between a pair of constituents i and j strongly depends on the relative positions of the constituents (for example, a nearest neighbor coupling in atomic systems) and cannot be defined universally.

In contrast, an universal coupling can be defined in the systems in which everyone talks to everyone at an equal strength. Such is the property of Bose-Einstein condensates (BECs) where all the constituents are in a common quantum state. In particular, the black holes represent such condensates of gravitons. For such a system we can define a very useful parameter,

$$\lambda \equiv N\alpha, \tag{1}$$

which is somewhat analogous to the 't Hooft coupling for gauge theories with N -colors [9]. Despite the crucial difference that in our case N is not an input of the theory, but rather a characteristic of a particular BEC, we shall refer to λ as the 't Hooft coupling.

This parameter plays the central role in our considerations since it determines how close is the system from *quantum* criticality. Thus, level of classicality of the system is not determined by only how large N is, but most importantly how far it is from the critical value $N = 1/\alpha$. This is the fundamental difference between black holes and other macroscopic bodies with many constituents. For the ordinary macroscopic objects, such as planets, the analog of the 't Hooft's coupling either cannot be defined or it is far from quantum criticality. This is why the ordinary macroscopic objects can be treated classically with a very good approximation, without encountering any seeming paradoxes. Contrary, as we have shown, black holes are always at the quantum critical point $N\alpha = 1$ up to $1/N$ corrections. As a result, black holes can never be treated classically. There are certain quantum effects (such as mass gap and degeneracy of Bogoliubov modes) that for large black holes become extremely important. In particular, at the quantum critical point small subsystems are maximally entangled i.e the entanglement entropy for the reduced one particle density matrix is maximal.

We thus, have outlined the following sequence of macroscopic systems with increasing level of quantumness:

Ordinary macroscopic objects (e.g., planets or buckets of water).

Quantum Characteristics : N exists, λ cannot be defined.

↓

Generic (non-critical) Bose-Einstein-Condensates.

Quantum Characteristics : Both N and λ are well defined, but $\lambda \neq 1$.

↓

Black holes: Bose-Einstein condensates stuck at the quantum critical point.

Quantum Characteristics : Both N and λ are well defined, and $\lambda = 1$.

In order to explain this profound difference, let us consider a hypothetical gravitating source of the mass of a neutron star. We shall use an oversimplified model in which we shall approximate the source by a collection of N_B particles of baryonic mass, $m_B \sim \text{GeV}$, stabilized by some non-gravitational forces. We shall ignore the contribution to the energy from the stabilizing force. Then by dialing the strength of the stabilizing force, we can bring the system to the critical point of black hole formation. In the classical approximation such a "neutron star" outside produces a gravitational field identical to the one produced by a classical Schwarzschild black hole. So why is the case that for the neutron star the quantum effects are not important whereas for a black hole of the same mass they are absolutely crucial?

In order to answer this question let us reduce the quantum portrait of the above system to its bare essentials. We are dealing with a source, represented by a multi-baryon state of occupation number $N_B \sim 10^{57}$ and size $L \sim 10^6$ cm. This source is not a Bose-Einstein condensate, since baryons (even if spin-0) are not in the same state, and in particular their de Broglie wavelengths are much shorter than the size of the system. However, these baryons source gravity and produce gravitational field that contains approximately $N \sim 10^{77}$ gravitons. The two occupation numbers are related as,

$$N = N_B^2 (m_B/M_P)^2, \quad (2)$$

where M_P is the Planck mass, and we shall also define the Planck length $L_P \equiv \hbar/M_P$. Unlike baryons, these gravitons are much closed to being a

Bose-Einstein condensate, because the majority of them occupy the same state, and in particular have comparable characteristic wave-lengths L given by the size of the baryonic source, $L \sim L_{star}$. Due to this, in contrast to the baryonic constituents of the star, for gravitons we can define an universal quantum coupling,

$$\alpha \equiv (L_P/L)^2 \quad (3)$$

and the corresponding 't Hooft's coupling λ given by (1). The only caveat is that the graviton condensate is not self-sustained as long as $L_{star} > r_g$. That is, the gravitational mass (self-energy) of the graviton condensate $M_{gr} = N\hbar/L$ is below the mass of the baryonic source $M_{star} = N_B m_B$ and alone is not enough to keep the gravitons together. Classically, we think of this situation as the size of the source L_{star} being larger than the corresponding gravitational radius $r_g \equiv M_{star} L_P^2 / \hbar$, but we see that the quantum-mechanical reason is that the 't Hooft coupling is far from criticality. Indeed, expressing N and α through their dependence on L and r_g , we have,

$$\lambda \equiv N\alpha = (r_g/L)^2 = (r_g/L_{star})^2. \quad (4)$$

Thus, the classical statement that a given source is not a black hole ($r_g < L_{star}$), quantum-mechanically translates as the condition that the 't Hooft coupling of graviton condensate is weak, $\lambda < 1$. Thus, the standard semi-classical expansion in powers of r_g/L is nothing but an expansion in the 't Hooft coupling λ . This expansion ignores additional $1/N$ -effects. That is, it represents a planar approximation:

$$\lambda = \text{fixed}, \quad N = \infty. \quad (5)$$

Such approximation is justified *only* as long as λ is sub-critical.

Now imagine that by changing the parameters of the model (say, by decreasing a stabilizing force) we bring the source to the point $L_{star} = r_g$. Classically, we think of this point as a point of classical black hole formation, but in reality this is a critical point of a *quantum* phase transition! As we have shown [3], there are dramatic quantum effects which take place at this point. In particular, of order N Bogoliubov modes of the graviton condensate become gapless and nearly degenerate. The condensate starts a quantum depletion, leakage and a subsequent collapse. This is the underlying quantum-mechanical nature of the process that semi-classically is viewed as Hawking evaporation. But, Hawking's semi-classical limit in our language

corresponds to planar limit, in which only λ -corrections are kept whereas $1/N$ -corrections are not taken into the account. In reality every act of emission differs from this idealized approximation by $1/N$ -corrections. Our point is to stress the extreme importance of these corrections.

In other words for a generic BEC the quantity $1/N$ measures the *quantum noise* of the system. For $N \gg 1$ these effects can be thought as very tiny and effectively negligible. This is in fact the case provided the constituents of the system are not entangled. However, and this is the key of the quantum phase transition, quantum noise makes a dramatic difference when the constituents are maximally entangled i.e at the quantum critical point. In fact at this point the entanglement entropy for the reduced one particle density matrix becomes maximal and a new branch of light Bogoliubov modes appear [10]. This is something completely alien to any classical system. In this sense black holes are intrinsically quantum objects. This phenomenon is fully missed in classical or semi-classical analysis. Its discovery requires a microscopic quantum view.

Thus, even macroscopic black holes are quantum.

This is a very general message we wanted to bring across in this short note.

2 Quantumness Versus Semi-Classicality

Can the quantum effects we are pointing out be somehow read off in the standard semi-classical treatment? We shall now explain why the answer is negative.

In standard treatment the black holes are introduced through the metric $g_{\mu\nu}(x)$, which is an intrinsically-classical entity. The effects of quantum gravity are then thought to be accounted in terms of quantum corrections to metric, without abandoning the very concept of the (classical) metric. In other words, both before and after the quantum corrections the metric itself is treated as a background classical field. The role of the quantum gravity is reduced to understanding the rules of corrections according to which this classical entity changes, without abolishing the very concept of a background metric. We claim that for certain macroscopic systems, such as black holes, the above treatment is inconsistent.

It is absolutely crucial to understand that $1/N$ -corrections are intrinsically-quantum and can never be recast in form of some quantum-back-reacted

metric. Instead the very notion of the metric needs to be abandoned and be treated as approximate. In order to explain this, let us go through the three levels of quantumness:

$$\begin{array}{c}
\textbf{Classical: } \hbar = 0, \frac{1}{N} = 0; \\
\downarrow \\
\textbf{Semi-Classical: } \hbar \neq 0, \frac{1}{N} = 0 ; \\
\downarrow \\
\textbf{Quantum: } \hbar \neq 0, \frac{1}{N} \neq 0 .
\end{array}$$

Consider a light test body and a heavy source of energy momentum tensors $\tau_{\mu\nu}$ and $T_{\mu\nu}$ respectively. In classical GR a scattering of a probe on a source can be understood in terms of a propagation of the former in a background classical metric created by the latter, with an amplitude,

$$A_{Cl} = \int_x g_{\mu\nu}(x) \tau^{\mu\nu}(x), \quad (6)$$

where integration is performed over a four-dimensional space-time volume. The metric $g_{\mu\nu}$ is obtained by solving the classical Einstein equation with the source $T_{\mu\nu}$. It is well-known that exactly the same amplitude can be reproduced by summing up the infinite series of tree-level Feynman diagrams with intermediate graviton lines,

$$\begin{aligned}
A = & G_N \int_{x,y} T(x) \Delta(x-y) \tau(y) + \\
& G_N^2 \int_{x,y,z,w} T(x) T(y) \Delta(x-w) \Delta(y-w) \Delta(z-w) O(w) \tau(z) + \dots
\end{aligned} \quad (7)$$

Here $\Delta(x)$ is a graviton propagator, and tensorial indexes are suppressed. These series are non-zero despite the fact that we are working in $\hbar = 0$ limit, and they fully reproduce the result obtained by considering the motion in the classical metric (6). In fact, order by order the above series reproduce the expansion of a classical solution of Einstein equation in series of G_N . For example, for a spherical source of mass M , the above series reproduce the expansion of Schwarzschild metric in series of $\frac{r_g}{r}$ where $r_g \equiv 2G_N M$ is the gravitational radius of the source and r is a radial coordinate [11].

Let us now move towards the quantum picture, $\hbar \neq 0$. The standard idea about how to take into the account quantum gravity effects is to integrate out loops and write down the \hbar -corrected effective action for $g_{\mu\nu}(x)$. The action obtained in this way will in general contain an infinite series of curvature invariants, with each power of curvature being accompanied by a factor of L_P^2 (in absence of other input scales). The effective quantum-corrected metric is then represented as a solution to the equations obtained by varying the effective action. In this philosophy, the quantum gravity effects are accounted in form of a back reaction to the classical metric. The quantum-corrected metric obtained in such a way, although formally includes \hbar -effects is still treated as a classical entity:

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu} = g_{\mu\nu}(x) + \delta g_{\mu\nu}(x, \hbar). \quad (8)$$

In particular, the quantum-corrected scattering of a probe over the source in this limit can still be reduced to the effects of propagation in the background metric obtained by replacing in (6) $g_{\mu\nu}$ by $g'_{\mu\nu}$.

As a result, such an analysis is not really quantum, but rather *semi-classical*, as it never resolves the quantum constituents of the metric ($1/N = 0$).

This is the essence of semi-classical approximation: It reduces quantum effects to the \hbar -correction of classical entities, without resolving their constituency.

We thus claim, that the above treatment of quantum gravity misses out the $1/N$ -corrections, which are absolutely crucial for black holes. The physics generated by these corrections, is impossible to be reproduced by any quantum corrections to the classical metric. Instead, the very notion of the metric must be abandoned and only treated as approximate.

In our language it is clear why this is the only consistent treatment. Indeed, it is impossible to keep all three quantities M, L_P and \hbar finite, and simultaneously keep $1/N = 0$. Putting it differently, r_g -corrections are corrections in terms of series in 't Hooft coupling $\lambda = (\alpha N)$, which are different from $1/N$ -series. Naively, it seems that one can consistently keep the former while discarding the latter by taking the planar limit (5). However, this is an illusion, since in this limit also the black hole evaporation time (which scales as $N^{3/2}$) becomes infinite, so that the integrated effect is still finite.

Notice, that $1/N$ corrections are present already in the tree-level scattering of a probe over a black hole and come from the processes in which the

probe exchanges the momentum with individual constituent of the graviton condensate. Because the condensate is at the quantum critical point, such exchanges cost $1/N$ as opposed to e^{-N} .

The resulting quantum scattering amplitude A_Q differs from its classical counterpart by $1/N$ -effects,

$$A_Q = A_{Cl} + \mathcal{O}(1/N). \quad (9)$$

However, the crucial point is that, unlike the semi-classical case, these effects cannot be recast in form of propagation in any new corrected metric. That is, the quantum amplitude A_Q does not admit any representation in form of

$$A_Q = \int_x g_{\mu\nu}(x)' \tau^{\mu\nu}, \quad (10)$$

where $g_{\mu\nu}(x)'$ could be any sensible metric. Such representation of the amplitude ceases to exist as soon as we correctly account for $1/N$ -effects.²

3 $1/N$ -Corrections Account for Information

Obviously, the $1/N$ -corrections to semi-classical results are much stronger than the naively-expected e^{-N} -correction. However, from the first glance these enhanced corrections still look very small. This smallness is an illusion and in reality $1/N$ -corrections are precisely what one needs for the correct accounting of information-retrieval in black hole decay.

The reason is that $1/N$ -corrections to planar results are taking place for each act of emission. Over a black hole half-lifetime this deviation accumulates to order-one effect, which is sufficient to start resolving the information at order-one rate. As we have shown [2], this reproduces Page's time [12], which automatically follows from our picture.

It is crucial that N is not a fixed characteristics of the theory (unlike in gauge theories with N -colors) but rather a characteristic of a particular black hole. Moreover, it is a good characteristic only during the time $\sim \sqrt{N}L_P$, during which the black hole depletes and leaks decreasing N by one unit. This process continues self-similarly

$$N \rightarrow N - 1 \rightarrow N - 2 \dots \quad (11)$$

²The quantity N defined in [1] as a measure of classicality also emerges in [14]. However, there this quantity is unrelated to any quantum resolution of the constituents of the metric.

Each elementary step of the cascade reveals a distinct feature (information) encoded in a $1/N$ -suppressed deviation from the Hawking's idealized semi-classical result. To resolve this feature immediately is extremely improbable, but this is not an issue. Unitarity does not require the information to be resolvable immediately. It only requires that information is resolvable on the time-scale of black hole evaporation.

This is exactly the case, since probability to recognize the given feature over the black hole half-lifetime, which scales as $\sim N^{3/2}L_P$, is of order one. In other words, the increase of N suppresses the probability of decoding a given feature per emission time as $N^{-3/2}$, but correspondingly the black hole life-time increases as $N^{3/2}$, so that the product is always of order one. As a result, for arbitrarily large N the information starts to be recognizable at order-one rate after a half-lifetime of a black hole.

To reiterate the picture, let us imagine a situation when Alice is observing evaporation of a solar mass black hole. For simplicity, we shall exclude all non-gravitational species from the theory. Then from our point of view, such a black hole is a BEC of approximately kilometer wavelength gravitons of occupation number $N \sim 10^{76}$, with $\sim N$ gapless Bogoliubov modes. From the point of view of the quantum information this black hole is a message encoded in a $N \sim 10^{76}$ long sequence of 0-s and 1-s,

$$BH = (0, 0, 1, 0, 1, 1, 1, \dots), \quad (12)$$

where, the sequence is determined by the state of Bogoliubov modes.

After every time interval of approximately $\Delta\tau \sim \sqrt{N}L_P \sim 10^{-5}$ sec the message emits a graviton and becomes shorter by one unit. In the semi-classical (planar) approximation (5) Alice thinks that she sees a thermal evaporation of a black hole with a featureless (exactly thermal) spectrum. However, in reality she sees a depletion and leakage of graviton BEC, with features encoded in sub-leading $1/N$ -corrections. As we know [1,2], this correction to the black hole rate goes as $\Gamma_{feature} \sim N^{-3/2}L_P^{-1}$. Thus, probability for Alice to recognize the feature per emission time is $\Delta P = \Gamma_{feature}\Delta\tau \sim 1/N$. For a solar mass black hole this probability is 10^{-71} and is tiny. However, the time-scale available for Alice to resolve the feature is also enormous, and is given by the black hole life-time $\tau = N\Delta\tau \sim N^{3/2}L_P \sim 10^{73}$ sec! The probability to resolve the feature during this time is

$$P \sim \Delta P N \sim 1. \quad (13)$$

Notice, that by then Alice has witnessed $\sim N$ acts of emission and had of order $\tau \sim N^{3/2} L_P$ time for analyzing each of them. Consequently she accumulated order one knowledge about roughly the half of the structure of the message. This knowledge brings her to the point starting from which she begins to resolve information with order one probability.

It is important to stress that we are not modifying Hawking's entanglement at each step of the emission process by a factor $\frac{1}{N} \ln 2$. This would not do the job of reproducing Page's time [13]. What we are instead doing is to use $1/N$ effects (at the quantum critical point) to trigger depletion of *one* bit of information with probability $1/N$ in *each step* of the evaporation process.

This completes our point of nullifying the information paradox. Notice, that increasing N is not changing the final answer, since although it suppresses the feature per emission time, it also increases the available time for resolving it so that the two effects always balance each other.

This analysis also makes clear the fundamental mistake in the standard semi-classical reasoning. If the features were suppressed by e^{-N} instead of $1/N$, Alice would have never had enough time for resolving these features, and the paradox would follow. It is now clear that this "paradox" was a result of our misconception about the quantum properties of macroscopic black holes.

In summary the Bose-Einstein condensate approach to the black hole information paradox lies on the following basic points:

- Black hole emission is due to quantum depletion triggered by quantum noise. This quantum emission is not based on any form of Hawking pair creation in the near horizon geometry. It is a perfectly unitary process with a rate determined by the microscopic dynamics of the condensate.
- This emission rate is modified by $1/N$ effects.
- In particular if we tag a subset of N_B quanta the rate of leakage of any form of information encoded in those quanta (as could be a baryon number of the black hole) goes like $\frac{N_B}{N^2}$. This in particular means that the black hole can successfully hide some information as its baryon number – or any other form of message encoded within the tagged quanta – but only until reaching the half-evaporation point. The *observable* prediction of this picture is $1/N$ hair. In case of baryon number

this hair can have observable effects for astrophysical black holes, that are mostly made out of baryons.

In this respect we need to stress the following. Of course, one could argue that a general belief that unitary quantum gravity should not result in information paradox implicitly assumes that some mechanism should purify Hawking radiation. However, an issue that has never been addressed previously is how this potential purification of Hawking radiation affects the *folk dictum* that in any consistent theory of gravity there are no global symmetries. We want to stress that in the Bose-Einstein portrait approach to the mechanism of information retrieval, gravity is perfectly consistent with global symmetries [2]. Obviously, how purification affects the *dictum* depends on the strength of the corrections used to purify the emitted quanta. Our $1/N$ -corrections revoke the *dictum*.

In short, semi-classicality breaks down whenever quantum noise $1/N$ -effects become significant. This is unavoidably the case at the quantum phase transition point. The black hole emits as a normal quantum system, but its identity card is to be at a *quantum phase transition point*.

Finally, we wish to note on a possible avenue of probing the large N -picture. Recently, Veneziano [15] suggested a very interesting stringy computation that reveals $1/N$ -hair in string - string-hole scattering. Viewed as a black hole of occupation number $N = 1/g_s^2$ ($g_s \equiv$ string coupling), this result represents a manifestation of $1/N$ -hair suggested by black hole quantum N -portrait.

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